Superior edge bimagic labeling for connected and disconnected graphs

A. Amara Jothi[+], N.G. David[+] and J. Baskar Babujee*

[+]*Department of Mathematics*
Madras Christian College, Chennai-600 059
**Department of Mathematics*
Anna University, Chennai-600 025
Email: amarajothia@gmail.com, ngdmcc@gmail.com, baskarbabujee@yahoo.com.

Abstract: *An edge magic total labeling of a graph G(V, E) with p vertices and q edges is a bijection f : V(G)∪E(G) → {1, 2, ..., p+q} such that f(u) + f(uv) + f(v) is a constant k for any edge uv in E(G). If there exist two constants k_1 and k_2 such that the above sum is either k_1 or k_2, it is said to be edge bimagic total labeling. A total edge-magic (bimagic) graph is called super edge-magic (bimagic) if f(V(G)) = {1, 2, ..., p} and it is called superior edge-magic (bimagic) if f(E(G)) = {1, 2, ..., q}. In this article we exhibit the superior edge magic and bimagic labeling for some connected and disconnected graphs.*

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1. Introduction:

All graphs considered in this article are finite, simple and undirected. A graph G with p vertices and q edges is called total edge magic if there is a bijection f : V(G) ∪ E(G) → {1, 2, .., p+q} such that f(u) + f(uv) + f(v) is a constant k for any edge uv in E(G). The original concept of total edge-magic graph is due to Kotzig and Rosa [6] and they called it as magic graph. A total edge-magic graph is called a super edge-magic if f(V(G)) = {1, 2, ..., p}. Edge bimagic total labeling was introduced by J. Baskar Babujee [1] and studied in [2]. A graph G with p vertices and q edges is called total edge bimagic if there exists a bijection f : V(G) ∪ E(G) → {1, 2, ..., p+q} and two constants k_1 and k_2 such that f(u) + f(uv) + f(v) is either k_1 or k_2 for any edge uv ∈ E(G). A total edge-bimagic graph is called super edge-bimagic if f(V(G)) = {1, 2, ..., p} and it is called superior edge bimagic if f(E(G)) = {1, 2, ..., q}.

Definition 1.1: A graph G with p vertices and q edges is called total edge magic if there is a bijection f : V(G) ∪ E(G) → {1, 2, .., p+q} such that f(u) + f(uv) + f(v) is a constant k for any edge uv in E(G).

Definition 1.2: [2] A bijection f : V(G) ∪ E(G) → {1, 2, .., p+q} is said to be super edge bimagic total labeling of G if there exists two constants k_1 and k_2 such that f(u) + f(uv) + f(v) = k_1 or k_2 for any edge uv in E(G) and f(V) = {1, 2, ..., p}.

Definition 1.3: [4] A graph G = (V, E) with p vertices and q edges has a superior edge magic total labeling if there is a bijective function f: V ∪ E→{1, 2,…, p+q} such that f(u) + f(uv) + f(v) is a constant k for any uv ∈ E(G), where $f(E(G))$ = {1, 2, …, q}. If f(u) + f(uv) + f(v) are all distinct for all uv ∈ E(G), then the graph is called superior edge antimagic total labeling.
A graph which admits superior edge magic (antimagic) total labeling is called superior edge magic (antimagic).

Definition 1.4: [3] If $G_1(p_1, q_1)$ and $G_2(p_2, q_2)$ are two connected graphs then the graph obtained by superimposing any selected vertex of G_2 on any selected vertex of G_1 is denoted by $G_1 \hat{o} G_2$. The resultant graph contains $p_1 + p_2 - 1$ vertices and $q_1 + q_2$ edges. In general, there are $p_1 p_2$ possibilities of getting $G_1 \hat{o} G_2$ from G_1 and G_2.

Definition 1.5: [7] An alternate triangular snake $A(T_n)$ is obtained from an even path $v_1, v_2, \ldots, v_{2k}$ by joining v_i and v_{i+1} to a new vertex u_i for i = 1, 3, …, 2k-1. That is, every alternate edge in the path is replaced by triangle C_3.

In this article, the graphs $C_n^+ \hat{o} (K_{1,m} + K_1)$, Friendship graph F_n, $\langle P_n: \overline{K}_m \rangle$, $A(T_n)$, $(P_2 + nK_1) \cup K_{1,m}$ and $nP_3 \cup K_{1,m}$ are shown to admit superior edge magic and bimagic labeling.

2. Superior edge magic and bimagic labeling for connected graphs

Theorem 2.1: There exists at least one graph G from the class $C_n^+ \hat{o} (K_{1,m} + K_1)$, when n is odd that admits superior edge bimagic total labeling.

Proof: Let $G = C_n^+ \hat{o} (K_{1,m} + K_1)$ be the graph with vertex set $V(G) = \{u_i, v_i, w_j : 1 \le i \le n, 1 \le j \le m\} \cup \{w^1\}$ and edge set $E(G) = E_1 \cup E_2 \cup E_3 \cup E_4 \cup E_5$ where $E_1 = \{u_iu_{i+1} : 1 \le i \le n-1\}$, $E_2 = \{u_iv_i : 1 \le i \le n\}$, $E_3 = \{v_1w_j : 1 \le j \le m\}$, $E_4 = \{w^1w_j : 1 \le j \le m\}$, $E_5 = \{u_1u_n, w^1v_1\}$. A bijective function f: $V(G) \cup E(G) \to \{1, 2, ..., 4n+3m+2\}$ is given below.

(i) For edges in E_1

For i = 1 to n-1; let $f(u_iu_{i+1}) = i$.

For i = 1 to n; when $i \equiv 1 (\mathrm{mod}\ 2)$, let $f(u_i) = 3(n+m+1) + \frac{n+1}{2} - \frac{i+1}{2}$, and

when $i \equiv 0 (\mathrm{mod}\ 2)$, let $f(u_i) = 4n+3m+3 - \frac{i}{2}$.

Therefore, for i = 1 to n-1, we have

when $i \equiv 1 (\mathrm{mod}\ 2)$, $f(u_i) + f(u_iu_{i+1}) + f(u_{i+1}) = (3(n+m+1) + \frac{n+1}{2} - \frac{i+1}{2}) + i + (4n+3m+3 - \frac{i+1}{2}) = 7n+6m+\frac{n+11}{2} = k_1$,

and

when $i \equiv 0 (\mathrm{mod}\ 2)$, $f(u_i) + f(u_iu_{i+1}) + f(u_{i+1}) = (4n+3m+3 - \frac{i}{2}) + i + (3(n+m+1) + \frac{n+1}{2} - \frac{i+2}{2}) = 7n+6m+\frac{n+11}{2} = k_1$.

(ii) For edges in E_2

For i = 1 to n; let $f(v_i) = 2n+3m+2+i$.

For i = 1 to n; when $i \equiv 1 (\mathrm{mod}\ 2)$, let $f(u_iv_i) = 2n+1 - \frac{i+1}{2}$, and

when $i \equiv 0 (\mathrm{mod}\ 2)$, let $f(u_iv_i) = n + \frac{n+1}{2} - \frac{i}{2}$.

Therefore, for i = 1 to n, we have

when $i \equiv 1 (\mathrm{mod}\ 2)$,

$$f(u_i) + f(u_iv_i) + f(v_i) = (3(n+m+1) + \frac{n+1}{2} - \frac{i+1}{2}) + (2n+1 - \frac{i+1}{2}) + (2n+3m+2+i) = 7n+6m+\frac{n+11}{2} = k_1, \text{ and}$$

when $i \equiv 0 (\mathrm{mod}\ 2)$, $f(u_i) + f(u_iv_i) + f(v_i) = (4n+3m - \frac{i}{2}) + (n + \frac{n+1}{2} - \frac{i}{2}) + (2n+3m+2+i) = 7n+6m+\frac{n+11}{2} = k_1$.

(iii) For edges in E_3

For j = 1 to m; let $f(w_j) = 2n+3m+3-j$, $f(v_1w_j) = 2n+j$. Hence we have,

$$f(w_j) + f(w_jv_1) + f(v_1) = (2n+3m+3-j) + (2n+j) + (2n+3m+3) = 6(n+m+1) = k_2.$$

(iv) For edges in E_4

For j = 1 to m; let $f(w^1w_j) = 2n+m+1+j$ and $f(w^1) = 2(n+m+1)$. Hence we have,

$$f(w_j) + f(w_jw^1) + f(w^1) = (2n+3m+3-j) + (2n+m+1+j) + (2(n+m+1)) = 6(n+m+1) = k_2.$$

(v) For edges in E_5

Let $f(w^1v_1) = 2n+m+1$, $f(u_1u_n) = n$. Then we have,

$$f(v_1) + f(w^1v_1) + f(w^1) = (2n+3m+3) + (2n+m+1) + 2(n+m+1) = 6(n+m+1) = k_2, \text{ and}$$

$$f(u_1) + f(u_1u_n) + f(u_n) = (3m+3n+\frac{n+1}{2}+2)+n+(3(n+m+1)) = 7n+6m+\frac{n+11}{2} = k_1.$$

We observe that there are two constants $k_1 = 7n+6m+\frac{n+1}{2}$ and $k_2 = 6(n+m+1)$ such that for each edge uv in E(G), f(u) + f(uv) + f(v) is either k_1 or k_2.

Hence the graph $G = C_n^{!}\ \hat{o}\ (K_{1,m}+K_1)$ admits superior edge bimagic total labeling.

Theorem 2.2: Friendship graph F_n, $n \geq 3$ admits superior edge bimagic total labeling.

Proof: Let $G = F_n$ be the Friendship graph with vertex set $V(G) = \{w, u_i, v_i : 1 \leq i \leq n\}$ and edge set $E(G) = E_1 \cup E_2 \cup E_3$ where $E_1 = \{wu_i : 1 \leq i \leq n\}$, $E_2 = \{v_iw : 1 \leq i \leq n\}$, $E_3 = \{u_iv_i : 1 \leq i \leq n\}$. We define a bijective function $f: V(G) \cup E(G) \rightarrow \{1, 2, ..., 5n+1\}$ as follows.

Case (i): n is odd

(i) For edges in E_1

For i =1 to n; let $f(u_i) = 3n+1+i$, $f(u_iw) = 3n+1-i$ and $f(w) = 3n+1$ then we have,

$$f(u_i) + f(u_iw) + f(w) = (3n+1+i)+(3n+1-i)+(3n+1) = 9n+3 = k_1.$$

(ii) For edges in E_2

For i = 1 to n; when $i \equiv 1 \pmod 2$, let $f(v_i) = 4n+\frac{n+1}{2}+2-\frac{i+1}{2}$, $f(v_iw) = 2n-\frac{n+1}{2}+\frac{i+1}{2}$, and

when $i \equiv 0 \pmod 2$, let $f(v_i) = 5n+2-\frac{i}{2}$, $f(v_iw) = n+\frac{i}{2}$.

Therefore, for i = 1 to n, we have

when $i \equiv 1 \pmod 2$, $f(v_i) + f(v_iw) + f(w) = (4n+2+\frac{n+1}{2}-\frac{i+1}{2})+(2n-\frac{n+1}{2}+\frac{i+1}{2})+(3n+1) = 9n+3 = k_1$, and

when $i \equiv 0 \pmod 2$, $f(v_i) + f(v_iw) + f(w) = (5n+2-\frac{i}{2})+(n+\frac{i}{2})+(3n+1) = 9n+3 = k_1$.

(iii) For edges in E_3

For i = 1 to n; when $i \equiv 1 \pmod 2$, let $f(v_iu_i) = n+1-\frac{i+1}{2}$, and

when $i \equiv 0 \pmod 2$, let $f(v_iu_i) = \frac{n+1}{2}-\frac{i}{2}$.

Then, for i = 1 to n, we have

when $i \equiv 1 \pmod 2$, $f(v_i) + f(v_iu_i) + f(u_i) = (4n+\frac{n+1}{2}+2-\frac{i+1}{2})+(n+1-\frac{n+1}{2})+(3n+1+i) = 8n+3+\frac{n+1}{2} = k_2$, and

when $i \equiv 0 \pmod 2$, $f(v_i) + f(v_iu_i) + f(u_i) = (5n+2-\frac{i}{2})+(\frac{n+1}{2}-\frac{i}{2})+(3n+1+i) = 8n+3+\frac{n+1}{2} = k_2$.

Case (ii): n is even

(i) For edges in E_1

For i =1 to n; let $f(u_i) = 3n+1+i$, $f(u_iw) = 3n+1-i$, $f(w) = 3n+1$ then we have,

$$f(u_i) + f(u_iw) + f(w) = (3n+1+i)+(3n+1-i)+(3n+1) = 9n+3 = k_1.$$

(ii) For edges in E_2

For i = 1 to n; when $i \equiv 1 \pmod 2$, let $f(v_i) = 5n+2-\frac{i+1}{2}$, $f(v_iw) = n+\frac{i+1}{2}$, and

when $i \equiv 0 \pmod 2$, let $f(v_i) = 4n+2+\frac{n}{2}+2-\frac{i}{2}$, $f(v_iw) = n+\frac{n}{2}+\frac{i}{2}$.

Then for i = 1 to n, we have

when $i \equiv 1 \pmod 2$, $f(v_i) + f(v_iw) + f(w) = (5n+2-\frac{i+1}{2})+(n+\frac{i+1}{2})+(3n+1) = 9n+3 = k_1$, and when $i \equiv 0 \pmod 2$, $f(v_i) + f(v_iw) + f(w) = (4n+\frac{n}{2}+2-\frac{i}{2})+(n+\frac{n}{2}+\frac{i}{2})+(3n+1) = 9n+3 = k_1$.

(iii) For edges in E_3

For $i = 1$ to n; when $i \equiv 1 \pmod 2$, $f(v_iu_i) = \frac{n}{2}+1-\frac{i+1}{2}$, and

when $i \equiv 0 \pmod 2$, $f(v_iu_i) = n-\frac{i}{2}$.

Then, for $i = 1$ to n, we have

when $i \equiv 1 \pmod 2$, $f(v_i) + f(v_iu_i) + f(u_i) = (3n+1+i)+(\frac{n}{2}+1-\frac{i+1}{2})+(5n+2-\frac{i+1}{2}) = 8n+3+\frac{n+1}{2} = k_2$, and

when $i \equiv 0 \pmod 2$, $f(v_i) + f(v_iu_i) + f(u_i) = (3n+1+i)+(n-\frac{i}{2})+(4n+\frac{n}{2}+2-\frac{i}{2}) = 8n+3+\frac{n}{2} = k_2$.

We observe that there are two constants k_1 and k_2 such that for each edge uv in E(G), f(u) + f(uv) + f(v) is either k_1 or k_2. If n is odd, then the constants are $k_1 = 9n+3$ and $k_2 = 8n+3+\frac{n+1}{2}$. If n is even, then the constants are $k_1 = 9n+3$ and $k_2 = 8n+3+\frac{n}{2}$. Hence the graph F_n $(n \geq 3)$ admits superior edge bimagic total labeling.

Definition: 2.3: Let $G = \langle P_n : \overline{K_m} \rangle$, $n \geq (2 \text{ or } 3)$ be a graph obtained by joining the two pendant vertices of the path P_n to m isolated vertices.

Theorem 2.4: When n is even the graph $G = \langle P_n : \overline{K_m} \rangle$, $n \geq 2$ admits superior edge magic labeling.

Proof: Let $G = \langle P_n : \overline{K_m} \rangle$ be the graph with vertex set $V = \{u_i, v_j : 1 \leq i \leq n, 1 \leq j \leq m\}$ and edge set $E = E_1 \cup E_2 \cup E_3$ where $E_1 = \{u_1v_j : 1 \leq j \leq m\}$, $E_2 = \{u_iu_{i+1} : 1 \leq i \leq n-1\}$, $E_3 = \{u_nv_j : 1 \leq j \leq m\}$. We define a bijective function f: $V(G) \cup E(G) \rightarrow \{1, 2, ..., 3m+2n-1\}$ as follows.

(i) For edges in E_1

For $j = 1$ to m; let $f(u_1v_j) = j$, $f(v_j) = 3m+n+\frac{n}{2}-j$ and let $f(u_1) = 3m+2n-1$. Then we have,

$$f(u_1) + f(v_j) + f(u_1v_j) = (3m+2n-1)+j+(3m+n+\frac{n}{2}-j) = 6m+\frac{7n}{2}-1 = k.$$

(ii) For edges in E_2

For $i = 1$ to $n-1$; let $f(u_iu_{i+1}) = m+i$ and let $f(u_n) = 2m+n$.

For $i = 1$ to $n-1$; when $i \equiv 1 \pmod 2$, let $f(u_i) = 3m+2n-1-\frac{i-1}{2}$, and

when $i \equiv 0 \pmod 2$, let $f(u_i) = 2m+n+\frac{n}{2}-\frac{i}{2}$.

Therefore, for $i = 1$ to $n-1$, we have

when $i \equiv 0 \pmod 2$, $f(u_i) + f(u_iu_{i+1}) + f(u_{i+1}) = (2m+n+\frac{n}{2}-\frac{i}{2})+(m+i)+(3m+2n-1-\frac{i}{2}) = 6m+\frac{7n}{2}-1 = k$, and

when $i \equiv 1 \pmod 2$, $f(u_i) + f(u_iu_{i+1}) + f(u_{i+1}) = (3m+2n-1-\frac{i-1}{2})+(m+i)+(2m+n+\frac{n}{2}-\frac{i+1}{2}) = 6m+\frac{7n}{2}-1 = k$.

(iii) For edges in E_3

For $j = 1$ to m; let $f(u_nv_j) = m+n+j-1$ then we have,

$$f(u_n) + f(u_nv_j) + f(v_j) = (2m+n)+(m+n-1+j)+(3m+n+\frac{n}{2}-j) = 6m+\frac{7n}{2}-1 = k.$$

We observe that there is a constant $k = 6m+\frac{7n}{2}-1$ such that for each edge uv in E(G), f(u) + f(uv) + f(v) = k.

Hence the graph $G = \langle P_n : \overline{K_m} \rangle$ admits superior edge magic total labeling.

Example 2.5: superior edge magic labeling of a graph $\langle P_{6:}\ \overline{K}_4 \rangle$ is shown in figure 1.

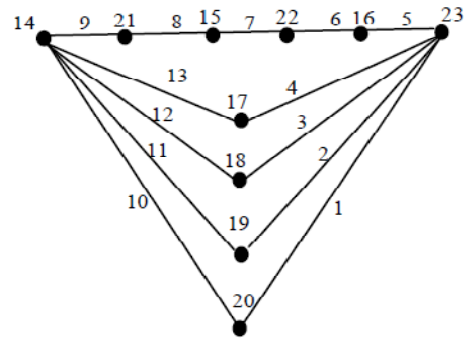

Fig 1: k_1 = 44

Theorem 2.6: When n is odd the graph $G = \langle P_n : \overline{K}_m \rangle$, $n \geq 3$ admits superior edge bimagic labeling.

Proof: Let $G = \langle P_n : \overline{K}_m \rangle$ be the graph with vertex set $V = \{u_i, v_j : 1 \leq i \leq n, 1 \leq j \leq m\}$ and edge set $E = E_1 \cup E_2 \cup E_3$ where $E_1 = \{u_1v_j : 1 \leq j \leq m\}$, $E_2 = \{u_iu_{i+1} : 1 \leq i \leq n\text{-}1\}$, $E_3 = \{u_nv_j : 1 \leq j \leq m\}$. We define a bijective function $f: V(G) \cup E(G) \rightarrow \{1, 2, ..., 3m+2n-1\}$ as follows.

(i) For edges in E_1

For j = 1 to m; let $f(u_1v_j) = m+n-1+j$, $f(v_j) = 3m+n+\frac{n-1}{2} - j$ and let $f(u_1) = 3m+n+\frac{n-1}{2}$. Then we have,

$$f(u_1) + f(u_1v_j) + f(v_j) = (3m+n+\frac{n-1}{2})+(m+n-1+j)+(3m+n+\frac{n-1}{2} - j) = 7m+4n-2 = k_1.$$

(ii) For edges in E_2

For i = 1 to n; let $f(u_iu_{i+1}) = m+n-i$.

For i = 1 to n; when $i \equiv 1 (\text{mod } 2)$, let $f(u_i) = 3m+n-1+\frac{n-1}{2}+\frac{i+1}{2}$, and

when $i \equiv 0 (\text{mod } 2)$, let $f(u_i) = 2m+n-1+\frac{i}{2}$.

Therefore, for **i** = 1 to n, we have

when $i \equiv 1$ (mod 2),

$$f(u_i) + f(u_iu_{i+1}) + f(u_{i+1}) = (3m+n-1+\frac{n-1}{2}+\frac{i+1}{2})+(m+n-i)+(2m+n-1+\frac{i+1}{2}) = 6m+\frac{7n-3}{2} = k_2, \text{ and}$$

when $i \equiv 0$ (mod 2),

$$f(u_i) + f(u_{i+1}) + f(u_iu_{i+1}) = (3m+2n-1+\frac{i}{2})+(m+n-i)+(3m+n-1+\frac{n-1}{2}+\frac{i+2}{2}) = 6m+\frac{7n-3}{2} = k_2.$$

(iii) For edges in E_3

For j = 1 to m; let $f(u_nv_j) = j$ and let $f(u_n) = 3m+2n\text{-}1$. Then we have,

$$f(u_n) + f(u_nv_j) + f(v_j) = (3m+2n-1)+j+(3m+n+\frac{n-1}{2} - j) = 6m+\frac{7n-3}{2} = k_2.$$

We observe that there are two constants $k_1 = 7m+4n-2$ and $k_2 = 6m+\frac{7n-3}{2}$ such that for each edge uv in E(G), f(u) + f(uv) + f(v) is either k_1 or k_2. Hence the graph $G = \langle P_n : \overline{K}_m \rangle$ admits superior edge bimagic total labeling.

Theorem 2.7: Alternate triangular snake graph $A(T_n)$, $n \geq 2$ admits superior edge bimagic total labeling.

Proof: Let $G = A(T_n)$ be the graph with vertex set $V = \{u_i, v_i, w_i : 1 \leq i \leq n\}$ and edge set $E = E_1 \cup E_2 \cup E_3 \cup E_4 \cup E_5$ where $E_1 = \{v_{n-i}w_{n-i} : 0 \leq i \leq n-1\}$, $E_2 = \{w_{n-i}v_{n-i+1} : 0 \leq i \leq n-1\}$, $E_3 = \{u_{n-i}w_{n-i} : 0 \leq i \leq n-2\}$, $E_4 = \{u_{n-i}v_{n-i}: 0 \leq i \leq n-2\}$ and $E_5 = \{u_1w_1, u_1v_1\}$. A bijective function $f: V(G) \cup E(G) \to \{1, 2, ..., 7n-1\}$ is given below.

(i) For edges in E_1

For i = 0 to n-1; let $f(v_{n-i}w_{n-i}) = 2i+1$, $f(v_{n-i}) = 6n-1-i$ and $f(w_{n-i}) = 7n-1-i$ then we have,

$f(v_{n-i}) + f(v_{n-i}w_{n-i}) + f(w_{n-i}) = (6n-1-i)+(2i+1)+(7n-1-i) = 13n-1 = k_1$.

(ii) For edges in E_2

For i = 0 to n-1; let $f(v_{n-i+1}) = 6n-i$ and $f(w_{n-i}v_{n-i+1}) = 2i$ then we have,

$f(w_{n-i}) + f(w_{n-i}v_{n-i+1}) + f(v_{n-i+1}) = 7n-1-i+2i+6n-i = 13n-1 = k_1$.

(iii) For edges in E_3

For i = 0 to n-2; let $f(u_{n-i}) = 5n-2-i$ and $f(u_{n-i}w_{n-i}) = 2n+2i+1$ then we have,

$f(u_{n-i}) + f(u_{n-i}w_{n-i}) + f(w_{n-i}) = (5n-2-i)+(2n+2i+1)+(7n-i-1) = 14n-2 = k_2$.

(iv) For edges in E_4

For i = 0 to n-2; let $f(u_{n-i}v_{n-i}) = 2(n+i+1)$ and we have,

$f(u_{n-i}) + f(u_{n-i}v_{n-i}) + f(v_{n-i}) = (5n-2-i)+(2(n+1+i)+(6n-i-1) = 13n-1 = k_1$.

(v) For edges in E_5

Let $f(u_1) = 5n-1$, $f(v_1) = 5n$, $f(u_1v_1) = 4n-1$, $f(w_1) = 6n$ and $f(u_1w_1) = 2n$. Then we have,

$f(u_1) + f(u_1w_1) + f(w_1) = (5n-1)+(2n)+(6n) = 13n-1 = k_1$, and

$f(u_1) + f(v_1) + f(u_1v_1) = 5n-1+5n+4n-1 = 14n-2 = k_2$.

We observe that there are two constants $k_1 = 13n-1$ and $k_2 = 14n-2$ such that for each edge uv in E(G), f(u) + f(uv) + f(v) is either k_1 or k_2. Hence the graph $G = A(T_n)$ admits superior edge bimagic labeling.

3. Superior edge bimagic labeling for disconnected graphs

Theorem 3.1: The graph $(P_2 + nK_1) \cup K_{1,m}$ $(n, m \geq 2)$ admits superior edge bimagic total labeling.

Proof: Let $G = (P_2 + nK_1) \cup K_{1,m}$ be a disconnected graph with vertex set $V(G) = \{ v_i : 1 \leq i \leq n\} \cup \{ u_j :1 \leq j \leq m \} \cup \{u^1, v^1, w^1\}$ and edge set $E(G) = E_1 \cup E_2 \cup E_3 \cup E_4$ where $E_1 = \{v^1v_i : 1 \leq i \leq n \}$, $E_2 = \{u^1v_i : 1 \leq i \leq n \}$, $E_3 = \{w^1 u_j : 1 \leq j \leq m \}$ and $E_4 = \{u^1v^1\}$. A bijective function $f: V(G) \cup E(G) \to \{1, 2, ..., 3n+2m+4\}$ is given bellow.

(i) For edges in E_1

For i = 1 to n; let $f(v_i) = 3(n+1)+m-i$, $f(v^1v_i) = i$ and let $f(v^1) = 3(n+1)+m$ then we have,

$f(v^1) + f(v^1v_i) + f(v_i) = (3(n+1)+m)+i +(3(n+1)+m-i) = 2(3n+m+3) = k_1$.

(ii) For edges in E_2

For i = 1 to n; let $f(u^1v_i) = n+1+i$ and let $f(u^1) = 2(n+1)+m$ then we have,

$f(u^1) + f(v_i) + f(u^1v_i) = (2(n+1)+m)+(n+1+i)+(3(n+1)+m-\mathbf{i}) = 2(3n+m+3) = \mathbf{k}_1$.

(iii) For edges in E_3

For j = 1 to m; let $f(w^1u_j) = 2n+1+\mathbf{j}$, $f(u_j) = 3n+5+2m-j$ and let $f(w^1) = 3n+m+4$ then we have,

$f(w^1) + f(w^1u_j) + f(u_j) = (3n+m+4)+(2n+1+j)+(3n+5+2m-j) = 8n+3m+10 = k_1$.

(iv) For edge u^1v^1 in E_4

Let $f(u^1 v^1) = n+1$ then we have $f(u^1) + f(u^1v^1) + f(v^1) = (2(n+1)+m)+(m+n+1)+(3(n+1)) = 2(3n+m+3) = k_1$.

We observe that there are two constants $k_1 = 2(3n+m+3)$ and $k_2 = 8n+3m+10$ such that for each edge uv in E(G), f(u) + f(uv) + f(v) is either k_1 or k_2. Hence the graph $(P_2 + nK_1) \cup K_{1,m}$ $(n, m \geq 2)$ admits superior edge bimagic total labeling.

Theorem 3.2: The graph $nP_3 \cup K_{1,m}$ ($n, m \geq 2$) admits superior edge bimagic total labeling.

Proof: Let $G = nP_3 \cup K_{1,m}$ be a disconnected graph with vertex set $V(G) = \{u_i, v_i, w_i : 1 \leq i \leq n\} \cup \{w^1\} \cup \{u_j^1 : 1 \leq j \leq m\}$ and edge set $E(G) = E_1 \cup E_2 \cup E_3$ where $E_1 = \{u_iv_i : 1 \leq i \leq n\}$, $E_2 = \{u_iw_i : 1 \leq i \leq n\}$, $E_3 = \{w^1, u_j^1 : 1 \leq j \leq m\}$. We define a bijective function $f: V(G) \cup E(G) \rightarrow \{1, 2, ..., 5n+2m+1\}$ as follows.

Case (i): n is odd

(i) For edges in E_1

For i = 1 to n; let $f(u_i) = 4n+2m+1+i$.

For i = 1 to n; when $i \equiv 1 (\bmod 2)$, let $f(v_i) = 3n+m+1-\frac{i+1}{2}$, $f(u_iv_i) = n+\frac{n+1}{2}+1-\frac{i+1}{2}$, and

when $i \equiv 0 (\bmod 2)$, let $f(v_i) = 4n+m+1-\frac{i}{2}$, $f(u_iv_i) = \frac{n+1}{2}-\frac{i}{2}$.

Therefore, for i = 1 to n, we have

when $i \equiv 1 (\bmod 2)$,

$$f(u_i) + f(u_iv_i) + f(v_i) = (4n+2m+1+i)+(n+\frac{n+1}{2}+1-\frac{i+1}{2})+(3n+m+1-\frac{i+1}{2}) = 8n+3m+2+\frac{n+1}{2} = k_1, \text{ and}$$

when $i \equiv 0 (\bmod 2)$, $f(u_i) + f(u_iv_i) + f(v_i) = (4n+2m+1+i)+(\frac{n+1}{2}-\frac{i}{2})+(4n+m+1-\frac{i}{2}) = 8n+3m+2+\frac{n+1}{2} = k_1$.

(ii) For edges in E_2

For i = 1 to n; when $i \equiv 1 (\bmod 2)$, $f(u_iw_i) = n+1-\frac{i+1}{2}$, $f(w_i) = 2n+m+\frac{n+1}{2}-\frac{i}{2}$ and

when $i \equiv 0 (\bmod 2)$, let $f(u_iw_i) = 2n+1-\frac{i}{2}$, $f(w_i) = 3n+m+\frac{n+1}{2}-\frac{j+1}{2}+1$.

Therefore, for i = 1 to n, we have

when $i \equiv 1 (\bmod 2)$,

$$f(u_i) + f(u_iw_i) + f(w_i) = (4n+2m+1+i)+(n+1-\frac{i+1}{2})+(3n+m+1+\frac{n+1}{2}-\frac{i+1}{2}) = 8n+3m+2+\frac{n+1}{2} = k_1, \text{ and}$$

when $i \equiv 0 (\bmod 2)$,

$$f(u_i) + f(u_iw_i) + f(w_i) = (4n+2m+1+i) + (2n+1-\frac{i}{2})+(2n+m+\frac{n+1}{2}-\frac{i}{2}) = 8n+3m+2+\frac{n+1}{2} = k_1.$$

(iii) For edges in E_3

For j = 1 to m; let $f(u_j^1) = 4n+2m+2-j$, $f(w^1u_j^1) = 2n+j$ and let $f(w^1) = 4n+m+1$. Then we have,

$f(w^1) + f(w^1u_j^1) + f(u_j^1) = (4n+m+1)+(2n-j)+(2n+4n+2m+j) = 10n+3m+3 = k_2$.

Case (ii): n is even

(i) For edges in E_1

For i = 1 to n; let $f(u_i) = 4n+2m+1+i$.

For i = 1 to n; when $i \equiv 1 (\bmod 2)$, let $f(v_i) = 3n+2m+1+i$, $f(u_iv_i) = 2n+1-2i$, and

when $i \equiv 0 (\bmod 2)$, let $f(v_i) = 4n+m+i$, $f(u_iv_i) = 2n+2-2i$.

Therefore, for i = 1 to n, we have

when $i \equiv 1 (\bmod 2)$, $f(u_i) + f(u_iv_i) + f(v_i) = (4n+2m+1+i)+(2n+1-2i)+(3n+2m+1+i) = 9n+4m+3 = k_1$, and

when $i \equiv 0 (\bmod 2)$, $f(u_i) + f(u_iv_i) + f(v_i) = (4n+2m+1+i)+(2n+2-2i)+(2n+m+i) = 8n+3m+2 = k_2$.

(ii) For edges in E_2

For i = 1 to n; when $i \equiv 1 (\bmod 2)$, let $f(u_iw_i) = 2n+2-2i$, $f(w_i) = 2n+m++i$, and

when $i \equiv 0 (\bmod 2)$, let $f(u_iw_i) = 2n+1-2i$, $f(w_i) = 3n+2m+i+1$.

Therefore, for i = 1 to n, we have

when $i \equiv 1 (\mathrm{mod}\ 2)$, $f(u_i) + f(u_i w_i) + f(w_i) = (4n+2m+1+i)+(2n+2-2i)+(2n+m+i) = 8n+3m+3 = k_2$, and

when $i \equiv 0 (\mathrm{mod}\ 2)$, $f(u_i) + f(u_i w_i) + f(w_i) = (4n+2m+1+i)+(2n-2i+1)+(3n+2m+1-i) = 9n+4m+3 = k_1$.

(iii) For edges in E_3

For j = 1 to m; let $f(u_j^1) = 3n+2m+2 - j$, $f(w^1 u_j^1) = 2n+j$ and let $f(w^1) = 3n+m+1$. Then we have,

$f(w^1) + f(w^1 u_j^1) + f(u_j^1) = (3n+m+1)+(2n+j)+(3n+2m+2-j) = 8n+3m+3 = k_2$.

We observe that there are two constants k_1 and k_2 such that for each edge uv in E(G), f(u) + f(uv) + f(v) is either k_1 or k_2. If n is odd, then the constants are $k_1 = 8n+3m+2+\frac{n+1}{2}$ and $k_2 = 10n+3m+3$.

If n is even, then the constants are $k_1 = 9n+4m+3$ and $k_2 = 8n+3m+3$. Hence the graph $nP_3 \cup K_{1,m}$ $(n, m \geq 2)$ admits superior edge bimagic total labeling.

Example 3.3: Superior edge bimagic total labeling of $3P_3 \cup K_{1,5}$ is shown in fig 2

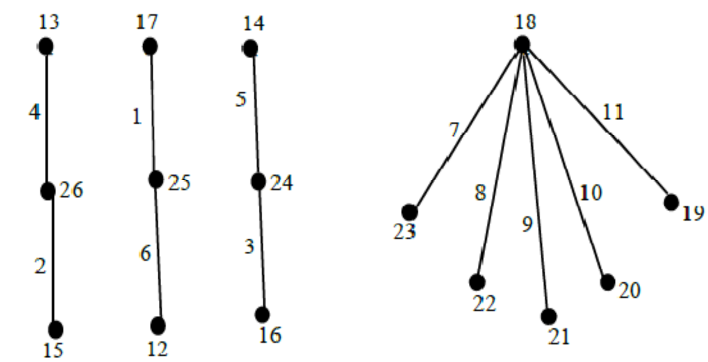

Fig 2: $k_1 = 43$, $k_2 = 48$

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