Abstract—The paper presents a new wavelet damage detection technique for structural health monitoring. A three dimensional plot of wavelet coefficients plotted in scale-translation plane provide necessary detection and localization of structural damage by showing high wavelet coefficients at the damage location. Damage is defined as the ratio of depth of cut to total height of beam ($c/h$) and numerically simulated by reducing the stiffness of the assumed element. The paper addresses the novel method to detect the smallest damage using different spatial inputs (mode shape and modal strain energy) on damage identification. The proposed method is numerically evaluated on a simple finite element beam model. The results of analysis indicate that the proposed continuous wavelet transform based damage identification method effectively identify single as well as multiple damages. Hence, it is shown that proposed method has the potential to identify damage in structures.

Keywords---Damage Detection, Spatial Data, Wavelets, Condition Monitoring

I. INTRODUCTION

DAMAGE in a mechanical (or) structural system may be contributed by various factors, such as excessive response, accumulative crack growth, wear and tear of working parts, and impact by a foreign object. Structural Health Monitoring (SHM) has emerged as a reliable, efficient and economical approach to monitor the system performance, detect such damage, assess/diagnose the structural health condition, and make corresponding maintenance decisions; consequently, structural safety and functionality will be significantly improved and a condition based maintenance procedure can be developed. Due to localization of damage in structures techniques using global averaging procedures, changes in eigen frequencies are less sensitive to initial or small changes. Hence techniques that process the local information based on wavelets have emerged recently. An application of spatial wavelet theory to damage identification in structures was proposed by Liew and Wang [1]. They calculated the wavelet coefficients along the length of the beam based on the numerical solution for the deflection of the beam, the damage location was then indicated by a peak in the coefficients of the wavelets along the length of the beam. Wang and Deng [2] described a method for detecting the location of localized defects. Quek et al. [3] also used wavelet analysis for crack identification in beams under both simply supported and fixed-fixed boundary conditions. Hong et al. [4] used the Lipschitz exponent for the detection of singularities in beam modal data. The Mexican hat wavelet was used and the damage extent has been related to different values of the exponent. The correlation, however, of the damage extent to the Lipschitz exponent is sensitive to both sampling distance and noise resulting in limited accuracy of the prediction. Recently, an interesting comparison between a frequency-based and mode shape-based method for damage identification in beam like structure has been published by Kim et al. [5]. Later Chang and Chen [6] used spatially distributed signals by Gabor wavelet transform so that the distributions of wavelet coefficients could identify the damage position of a rectangular plate by showing a peak at the position of the damage which was very sensitive to the damage size. Also Abdo and Hori [10] made numerical study of the relation between damage characteristics and changes in the dynamic properties are presented. It is found the rotation of mode shape has the characteristic of localization at the damaged region even though the displacement modes are not localized.

Despite of the extensive studies of vibration analysis on damaged structures, only few effective and practical techniques are found for very small damaged identification. In this paper a study is carried out to investigate the influence of using the mode shapes as an input for the wavelets on damage identification for different scenario of damage cases.

II. CONTINUOUS WAVELET TRANSFORM (CWT)

The Continuous Wavelet Transform (CWT) is defined as the sum over all time of the signal function of time or space (infinite) multiplied by a scaled, shifted version of a wavelet function $\psi$. For a spatial signal,
Where, \( f(x) \) is the spatial input signal, and \( x \) being the spatial coordinate. The results of the CWT are wavelet coefficients \( W_f(u,s) \) that are a function of the scale \( s \) and position \( u \). Since the input is spatial signal the wavelet transform is called Spatial Wavelet Transform. In case of damage identification in beam structures, the input signal may be mode shape, modal strain energy or the forced vibration data, where \( x \) is length of beam or node (element) number.

To perform the CWT, a basic wavelet function must be selected from the existing wavelet families. The basic wavelet function, known as the “mother wavelet” \( \psi(x) \) is dilated by a value \( s \) and translated by the parameter \( u \).

The dilation (expansion or compression) and the translation yield a set of basis functions defined as

\[
\psi(s,u,x) = \frac{1}{\sqrt{s}} \psi \left( \frac{x-u}{s} \right)
\]

The translation parameter, \( u \), indicates the space (or time) position of the relocated wavelet window in the wavelet transform. Shifting the wavelet window along the space (or time) axis implies examining the signal \( f(x) \) in the neighborhood of the current window location. The scale parameter, \( s \), indicates the width of the wavelet window.

The wavelets coefficients defined in equation indicate how similar is the function \( f(x) \) being analyzed to the wavelet function \( \psi(s,u,x) \).

In terms of a selected mother wavelet function \( \psi(x) \), the continuous wavelet transform of a signal \( f(x) \) is defined as (Daubechies, 1992)

\[
W_f(s,u) = \frac{1}{\sqrt{s}} \int_{-\infty}^{\infty} f(x) \psi \left( \frac{x-u}{s} \right) dx
\]

The wavelet transform can detect and characterize transients (caused due to damage) in a spatial signal with zooming procedure across scales. The wavelet coefficients measures the variation of \( f(x) \) in a neighborhood of \( u \) whose size is proportional to \( s \). Sharp transients create large amplitude wavelet coefficients. Thus high wavelet coefficients \( W_f(s,u) \) at a particular point on the spatial signal detects and locates the damage (Level I and II identification).

III. NUMERICAL SIMULATION

For numerical simulations, aluminum beam with square cross section of dimensions 1200 x 20 x 20 mm with young’s modulus of 70GPa and density of 2700 kg/m\(^3\) is used (Hong et al., 2002). Modeling and modal analysis is performed in ANSYS 11.0. The length of beam is divided into 2400 one-dimensional elements which fix width of each element equal to 0.5 mm, approximates width of actual crack. Damage is simulated at the 1600th element which is at 400 mm from left end as shown in Figure 1(a). The damage (c/h) was varied from 0.1 to 0.8 to give nine different damage cases. The beam is free at both the ends.

![Fig. 1 Beam model and Damage geometry](image)

**Table 3.1:** First three natural frequencies for undamaged and all damage case

<table>
<thead>
<tr>
<th>Damage case (c/h)</th>
<th>Mode 1</th>
<th>Mode 2</th>
<th>Mode 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Undamaged</td>
<td>72.562</td>
<td>199.88</td>
<td>391.49</td>
</tr>
<tr>
<td>0.1</td>
<td>72.554</td>
<td>199.86</td>
<td>391.49</td>
</tr>
<tr>
<td>0.2</td>
<td>72.539</td>
<td>199.81</td>
<td>391.48</td>
</tr>
<tr>
<td>0.3</td>
<td>72.517</td>
<td>199.74</td>
<td>391.47</td>
</tr>
<tr>
<td>0.4</td>
<td>72.477</td>
<td>199.6</td>
<td>391.45</td>
</tr>
<tr>
<td>0.5</td>
<td>72.404</td>
<td>199.32</td>
<td>391.41</td>
</tr>
<tr>
<td>0.6</td>
<td>72.238</td>
<td>198.71</td>
<td>391.33</td>
</tr>
<tr>
<td>0.7</td>
<td>71.745</td>
<td>196.99</td>
<td>391.08</td>
</tr>
<tr>
<td>0.8</td>
<td>69.824</td>
<td>190.8</td>
<td>390.21</td>
</tr>
<tr>
<td>0.9</td>
<td>55.789</td>
<td>163.38</td>
<td>386.45</td>
</tr>
</tbody>
</table>
IV. WAVELET ANALYSIS

The Spatial signal (fundamental mode shapes/ modal strain energy) from damaged beam is wavelet transformed using Wavelet Toolbox available in MATLAB 6.5. After some experimentation it is found that scales of 8 to 32 provided better results. The mother wavelet selected is Gaussian wavelet with four vanishing moments. The resulting wavelet coefficients are used in damage

V. RESULTS AND DISCUSSION

![Fig. 2 Damage case c/h=0.9](image1)

Fig. 2 Damage case c/h=0.9 (a) Fundamental mode shape (b) 3-D Wavelet plot in Scale-translation plane

Figure 2(a) shows the plot of displacement mode shape for undamaged and damaged beam with damage severity of c/h equal to 0.9 with which it is practically impossible to locate damage. The mode shape obtained from damaged beam is wavelet transformed using Gaussian wavelet and the resulting wavelet coefficients are plotted in scale-translation plane as shown in Figure 2(b). It is observed clearly in the 3- dimensional plot, that at node 1600, marked change in wavelet coefficients occur with respect to adjacent element, indicative of damage. Similar plots for the damage case c/h of 0.7, 0.5 and 0.2 are shown in Figure 3, Figure 3.5 and Figure 3.6 respectively.

![Fig. 3 Damage case c/h=0.7](image2)

Fig. 3 Damage case c/h=0.7 (a) Fundamental mode shape (b) 3-D Wavelet plot in Scale-translation plane

It is observed that from Figure 3(a) for case of c/h 0.7 the mode shapes corresponding to undamaged and damaged beam are identical and locating damage becomes impossible. But when the mode shape from damaged beam is wavelet transformed and plotted in scale-translation (node number) plane, damage can be clearly located by high wavelet coefficients at 1600th element as shown in figure 3 (b).
Figure 4 (b) shows the 3-D wavelet plot for c/h 0.5, which display considerable value of wavelet coefficients in comparison to value at damaged region. This again is due to the deceased curvature change in mode shape at the damaged region compared to change at the middle of mode shape.

Figure 5 (b) shows the 3-D wavelet plot for c/h 0.2, which display considerable less value of wavelet coefficients in comparison to value at damaged region. This again is due to the deceased curvature change in mode shape at the damaged region compared to change at the middle of mode shape.

VI. DAMAGE IDENTIFICATION WITH DAMAGE AT TWO LOCATIONS

To investigate the effectiveness of the proposed method to locate and quantify multiple damages using only the fundamental mode shape obtained from damaged beam the same beam previously considered is used with varying damages at 800 and 1600th element. Figure 6(a) shows the fundamental mode shape obtained from beam with damage c/h=0.7 at 800 and 1600th element. Again, for this case it is practically impossible to locate damage just by observing the mode shapes.
VII. SMALL LEVEL DAMAGE LOCALIZATION USING MODAL STRAIN ENERGY

It has been found that the method of using displacement mode shape to locate damage is insensitive to damage with c/h less than 0.2.

Figure 7 shows the 3-D wavelet plot for the case of c/h=0.1 where it is practically impossible to locate damage by examining the points of high wavelet coefficients.

For lesser value of damage severity the effectiveness of using elemental modal strain energy data (output from ANSYS) as input to wavelet transform is investigated. Since it has been proved that modal strain energy is more sensitive to damage than mode shape (Stubbs et al., 1995), using wavelet transform of modal strain energy is much more sensitive for lower level of damage.

Fig. 8 Wavelet based Damage identification using modal strain energy for damage case c/h=0.1 (a) Fundamental modal strain energy(b) 3-D wavelet plot.

Figure 8 (a) shows the first modal strain energy for c/h of 0.1, in which the damage can be identified clearly by a peak at 1600th point. Wavelet transformed modal stain energy plotted in scale-element number plane, magnifies the variation of at the damage which helps in clear identification of damage.

VIII. CONCLUSION

The three dimensional wavelets plot has the potential to detect, locate and quantify single as well as multiple damages and also has the capability of pin pointing exact damage. Some of the important conclusions drawn based on wavelet transform applied to damage identification in beam structure are given below

1. It is observed that the methods of using change in damaged mode shape with respect to undamaged, as input to wavelet are able to identify single and multiple location damage clearly without the use of undamaged (baseline) data.
2. Detailed study showed that the localization and
quantification strongly dependent on following factors:

a. Input used- mode shape, modal strain energy modal data
b. Wavelet used and Scales selected
c. Location of damage.

3. Method based on modal strain energy as input to wavelet transform is more sensitive than using displacement mode shape and is able to locate damage with severity value as small as 0.05

4. The proposed method can be used in real time detection of damage when suitable measuring techniques which are able to pick up the perturbations caused by presence of damage are utilized to obtain mode shape and modal strain energy.

REFERENCES: